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On The BRST Formulation of Diffeomorphism Invariant 4D Quantum Gravity

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Abstract

In this note we give some remarks on the BRST formulation of a renormalizable and diffemorphism invariant 4D quantum gravity recently proposed by the author, which satisfies the integrability condition by Riegard, Fradkin and Tseytlin at the 2-loop level. Diffeomorphism invariance requires an addition of the Wess-Zumino action, from which the Weyl action can be induced by expanding around a vacuum expectation value of the conformal mode. This fact suggests the theory has in itself a mechanism to remove extra negative-metric states dynamically.

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Diffeomorphism invariance requires that 4D quantum gravity becomes 4th order derivative theory for gravity sector. We recently showed [1, 2] that 4th order actions, including the Wess-Zumino (WZ) action [3, 4], are uniquely determined by diffeomorphism invariance. Then, the theory also becomes renormalizable [1]. Especially, our model satisfies the integrability condition on the WZ action discussed by Riegard, Fradkin and Tseytlin [3], which is generalized by the author to the form that can be applied to higher loops. A problem in 4th-order theories is that there are extra negative-metric states. Thus, the unitarity becomes obscure [5]. In this paper we shall see that there is a posibility that diffeomorphism invariance also ensures the unitarity.

Here, we briefly explain how to realize diffeomorphism invariance. The details of the argument were discussed in our previous papers [1, 2]. Perturbation theory is defined by replacing the invariant measure with the measure defined on the background-metric. As a lesson from 2D quantum gravity [6]–[11], in order to preserve background-metric independence, or diffeomorphism invariance, we must add an action, S, which satisfies the WZ condition [12], as

$$Z = \int \frac{[d\phi]_{\hat{g}}[e^{-h}de^{h}]_{\hat{g}}[df]_{\bar{g}}}{\mathrm{vol(diff.)}} \exp[-S(\phi,\bar{g}) - I(f,g)] \ , \tag{1}$$

where f is a matter field and I is an invariant action. The metric is now decomposed as $g_{\mu\nu} = e^{2\phi}\bar{g}_{\mu\nu}$ and $\bar{g}_{\mu\nu} = (\hat{g}e^{h})_{\mu\nu}$, where $tr(h) = 0$ [11]. The measures of the metric fields are defined on the background-metric by the norms:

$$< d\phi, d\phi >_{\hat{g}} = \int d^4x \sqrt{\hat{g}} (d\phi)^2 \ , \tag{2}$$

$$< dh, dh >_{\hat{g}} = \int d^4x \sqrt{\hat{g}} \ tr(e^{-h}de^{h})^2 \ . \tag{3}$$

The general coordinate transformation, $\delta g_{\mu\nu} = g_{\mu\lambda}\nabla_\nu \xi^\lambda + g_{\nu\lambda}\nabla_\mu\xi^\lambda$, is expressed in 4 dimensions as

$$\begin{aligned} \delta\phi &= \frac{1}{4}\hat{\nabla}_\lambda \xi^\lambda + \xi^\lambda \partial_\lambda \phi \ , \\ \delta\bar{g}_{\mu\nu} &= \bar{g}_{\mu\lambda}\bar{\nabla}_\nu\xi^\lambda + \bar{g}_{\nu\lambda}\bar{\nabla}_\mu\xi^\lambda - \frac{1}{2}\bar{g}_{\mu\nu}\hat{\nabla}_\lambda\xi^\lambda \ , \end{aligned} \tag{4}$$

where $\bar{\nabla}_\lambda \xi^\lambda = \hat{\nabla}_\lambda\xi^\lambda$ is used. Under a general coordinate transformation, $\delta I = 0$, but the WZ action is not invariant. δS is proportional to the form of

conformal anomaly [13]. Diffeomorphism invariance is realized such that δS cancels anomalous contributions, U, which originates from the fact that the measures defined above are no longer invariant under the transformation, as

$$\delta Z = - < \delta S + U >= 0 \ . \tag{5}$$

More rigorously, consider the regularized 1PI effective action, $\Gamma_{\rm eff}$, of the combined theory, $\mathcal{I} = S + I$, and require $\delta\Gamma_{\rm eff} = 0$, which determines S uniquely.

In 2 dimensions we can take the conformal gauge $h^\mu{}_\nu = 0$, and hence 2D quantum gravity coupled to conformal matter can be described as a free conformal field theory [7]. Of course, in 4 dimensions, the combined theory $\mathcal{I} = S + I$ can not be described as a free theory. We must take into account interactions between the conformal mode and the traceless mode in the WZ action as well as self-interactions of the traceless mode, which are ruled by the background-metric independence for the traceless mode [2, 1]. Thus, we must generalize the idea of 2D quantum gravity based on conformal field theories to one based on diffeomorphism invariance. The original idea on this matter is given in a study of 2D quantum dilaton gravity [9, 10],[2] and then developed to 4D quantum gravity [2].

The conformal-mode dynamics of the WZ action in 4 dimensions has been discussed in refs. [4] in analogy to 2D quantum gravity. But, in their model there is no interactions for the traceless mode, because they considered the WZ action as a full effective action given after the traceless mode as well as matter fields are integrated out. From the viewpoint of full diffeomorphism invariance, it is inaccurate in 4 dimensions.

In this note we give the BRST formulation [15, 16] of diffeomorphism invariant quantum gravity. We first review the BRST formulation of 2D quantum gravity [18, 19]. We here emphasize that the nilpotence of the BRST transformation, which is equivalent to diffeomorphism invariance, is realized dynamically. We then show that similar considerations can apply to diffeomorphism invariant 4D quantum gravity. At the end we discuss the

[2] There is an analogy between the dilaton field φ defined in [9, 10] and the traceless mode in our 4D model. Unfortunately, this model is unrenormalizable in the perturbation of non-minimal coupling because φ is a dimensionless scalar in 2 dimensions so that there are many diffeomorphism invariant counterterms like $\varphi^n \partial^\mu \varphi \partial_\mu \varphi$. On the other hand the dynamics of the traceless mode is ruled by the background-metric independence for the traceless mode, itself, so that the model has almost no ambiguity.

possibility of how to remove the negative-metric states in the 4D model from the viewpoint of diffeomorphism invariance. Preliminary discussions on this matter have already given in [1].

Our curvature conventions are $R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu}$ and $R^{\lambda}{}_{\mu\sigma\nu} = \partial_\sigma \Gamma^{\lambda}{}_{\mu\nu} - \cdots$.

2D quantum gravity

Firstly, we briefly review the BRST formulation of 2D quantum gravity. The WZ action in two dimensions, what is called the Liouville action, is given by integrating the 2D conformal anomaly as [14]

$$S(\phi, \bar{g}) = \frac{a}{4\pi} \int d^2x \sqrt{\hat{g}} (\bar{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \bar{R}\phi) \ . \tag{6}$$

In 2 dimensions we can take the gauge condition $h^{\mu}{}_{\nu} = 0$ up to the zero mode. The gauge-fixed combined action then becomes

$$\mathcal{I} = \frac{1}{4\pi} \int d^2x \sqrt{\hat{g}} \Big[a \Big(\bar{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \bar{R}\phi \Big) + \mathcal{L}_{GF+FP} + \Lambda e^{\alpha\phi} \Big] + I_M(f, g) \ , \tag{7}$$

where I_M is an invariant matter action. The gauge-fixing term and the Faddeev-Popov (FP) ghost action are given by

$$\mathcal{L}_{GF+FP} = -iB_{\mu\nu} (\bar{g}^{\mu\nu} - \hat{g}^{\mu\nu}) + 2\bar{g}^{\mu\nu} b_{\mu\lambda} \bar{\nabla}_\nu c^\lambda \ , \tag{8}$$

where the reparametrization ghost c^μ is a contravariant vector. $B_{\mu\nu}$ and the anti-ghost $b_{\mu\nu}$ are covariant symmetric traceless tensors. In the following, $\mathcal{I}$ is considered as a "classical" action.

Consider N massless scalars as matter fields. Then, diffeomorphism invariance requires that the coefficient, a, must be [7]

$$a = \frac{1}{6}(25 - N) \ . \tag{9}$$

The BRST transformation is given by

$$\begin{aligned}
&\delta_{\mathbf{B}} \bar{g}_{\mu\nu} = i(\bar{g}_{\mu\lambda} \bar{\nabla}_\nu c^\lambda + \bar{g}_{\nu\lambda} \bar{\nabla}_\mu c^\lambda - \bar{g}_{\mu\nu} \hat{\nabla}_\lambda c^\lambda) \ , \\
&\delta_{\mathbf{B}} \phi = ic^\lambda \partial_\lambda \phi + i\frac{1}{2} \hat{\nabla}_\lambda c^\lambda \ , \\
&\delta_{\mathbf{B}} b_{\mu\nu} = B_{\mu\nu} \ , \qquad \delta_{\mathbf{B}} B_{\mu\nu} = 0 \ , \\
&\delta_{\mathbf{B}} c^\mu = ic^\lambda \partial_\lambda c^\mu \ ,
\end{aligned} \tag{10}$$

where $c^\lambda\bar{\nabla}_\lambda c^\mu = c^\lambda\partial_\lambda c^\mu$ due to the anti-commutativity of c^μ. Note that the h-dependence appears only in $\delta_B\bar{g}_{\mu\nu}$. These transformations are nilpotent; $\delta_B^2(\bar{g}_{\mu\nu},\ \phi,\ b_{\mu\nu},\ c^\mu) = 0$. Then, the gauge-fixing term and the FP ghost action can be written as $\mathcal{L}_{GF+FP} = -i\delta_{\mathbf{B}}\{b_{\mu\nu}(\bar{g}^{\mu\nu} - \hat{g}^{\mu\nu})\}$ [17], where $\delta_B\bar{g}^{\mu\nu} = -\bar{g}^{\mu\lambda}\bar{g}^{\nu\sigma}\delta_B\bar{g}_{\lambda\sigma}$.

The well-known form of the BRST transformation in 2D quantum gravity is given by integrating out over the $B_{\mu\nu}$ field. Hence, we obtain the following one:

$$\begin{aligned}
&\delta_{\mathbf{B}}\phi = ic^\lambda\partial_\lambda\phi + i\frac{1}{2}\hat{\nabla}_\lambda c^\lambda \ , \\
&\delta_{\mathbf{B}}b_{\mu\nu} = 2i\mathcal{T}_{\mu\nu} \ , \\
&\delta_{\mathbf{B}}c^\mu = ic^\lambda\partial_\lambda c^\mu \ .
\end{aligned} \tag{11}$$

Now, $h^\mu{}_\nu$ becomes non-dynamical and we can set $\bar{g}_{\mu\nu} = \hat{g}_{\mu\nu}$ in the expressions. To guarantee $\delta_B h^\mu{}_\nu = 0$, c^μ should satisfy the conformal Killing equation, $\hat{\nabla}^\mu c^\nu + \hat{\nabla}^\nu c^\mu - \hat{g}^{\mu\nu}\hat{\nabla}_\lambda c^\lambda = 0$. $\mathcal{T}_{\mu\nu}$ is the modified energy-momentum tensor, which is determined by using the equation of motion for the traceless mode and tracelessness of $b_{\mu\nu}$, as [3]

$$\begin{aligned}
\mathcal{T}_{\mu\nu} &= T_{\mu\nu} - \frac{1}{2}\hat{g}_{\mu\nu}T^\lambda{}_\lambda \\
&= -\frac{a}{2}\Big\{\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\hat{g}_{\mu\nu}\partial^\lambda\phi\partial_\lambda\phi - \Big(\hat{\nabla}_\mu\hat{\nabla}_\nu - \frac{1}{2}\hat{g}_{\mu\nu}\hat{\Box}\Big)\phi\Big\} \\
&\quad + \hat{\nabla}_{(\mu}c^\lambda b_{\nu)\lambda} + \frac{1}{2}c^\lambda\hat{\nabla}_\lambda b_{\mu\nu} - \frac{1}{2}\hat{g}_{\mu\nu}\hat{\nabla}^\lambda c^\sigma b_{\lambda\sigma} + T^M_{\mu\nu} \ .
\end{aligned} \tag{12}$$

Here, $T_{\mu\nu} = -\frac{2\pi}{\sqrt{\hat{g}}}\frac{\delta\mathcal{I}}{\delta\hat{g}^{\mu\nu}}$ is the energy-momentum tensor of the gauge-fixed combined theory (7) with $h^\mu{}_\nu = 0$. $T^M_{\mu\nu}$ is the energy-momentum tensor of N massless scalars. Since, in two dimensions, $\mathcal{T}^\lambda{}_\lambda = 0$, 2D quantum gravity can be expressed by conformal field theory.

The transformation (11) is now no longer nilpotent "classically". Using expression (12) and the conformal Killing equation of c^μ, we can show that $\delta_B^2 b_{\mu\nu}$ produces a non-vanishing quantity,

$$\delta_B\mathcal{T}_{\mu\nu} = i\frac{a}{4}\Big(\hat{\nabla}_\mu\hat{\nabla}_\nu - \frac{1}{2}\hat{g}_{\mu\nu}\hat{\Box}\Big)\hat{\nabla}_\lambda c^\lambda \ , \tag{13}$$

[3] Because of the tracelessness of $b_{\mu\nu}$, there is an ambiguity γ, which is that one can add $\gamma b_{\mu\nu}\hat{\nabla}_\lambda c^\lambda$ to the energy-momentum tensor for the bc-system. It is now fixed by the condition $\delta_B\mathcal{I}_{bc} = 0$.

which implies that the energy-momentum tensor, $\mathcal{T}_{\mu\nu}$, forms Virasoro algebra with central charges $6a$ classically [10]. It reflects that the action, $\mathcal{I}$, is not BRST-invariant: [4]

$$\delta_B \mathcal{I} = \frac{ia}{8\pi}\int d^2x \sqrt{\hat{g}}\bar{R}\hat{\nabla}_\lambda c^\lambda \ . \tag{14}$$

The BRST invariance of 2D quantum gravity is realized dynamically as follows. Quantum effects, namely, conformal anomalies give contributions to the central charge, $N-25$, so that the total of the central charge becomes $c_{tot} = 6a + N - 25$. Thus, the nilpotence of the BRST transformation is realized at the quantum level when a is given by equation (9).

Finally, we give brief comments on physical states in 2D quantum gravity. The presence of the $\frac{1}{2}\hat{\nabla}_\lambda c^\lambda$ term in the BRST transformation of ϕ implies that the asymptotic state of the conformal mode, $\phi|0>$, is not physical [16]. On the other hand, the BRST invariant state, naively defined by $e^{\alpha\phi}|0>$, where α is a real value determined by the BRST invariance, is not normalizable [8]. Furthermore, in 2 dimensions, the normalizable Hamiltonian eigenstates are not BRST invariant because they clearly do not satisfy the Hamiltonian constraint, $H = \mathcal{T}^0_{\ 0} = 0$. Thus, there is no gravitational degrees of freedom in 2D quantum gravity.

4D quantum gravity

In 4 dimensions the WZ action becomes 4th order, parametrized by three constants a, b and c in the form

$$\begin{aligned} S(\phi,\bar{g}) \ &= \ \frac{1}{(4\pi)^2}\int d^4x \Big[\sqrt{\hat{g}}\Big\{a\bar{F}\phi + 2b\phi\bar{\Delta}_4\phi + b\Big(\bar{G} - \frac{2}{3}\,\bar{\Box}\,\bar{R}\Big)\phi\Big\} \\ &\qquad -\frac{1}{36}(2a+2b+3c)\Big(\sqrt{g}R^2 - \sqrt{\hat{g}}\bar{R}^2\Big)\Big] \ , \end{aligned} \tag{15}$$

where F is the square of the Weyl tensor and G is the Euler density defined by

$$F \ = \ R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2 \ , \tag{16}$$

$$G \ = \ R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \ . \tag{17}$$

[4] Eq. (14) can be derived either by applying (10) to the action (7) or by applying (11) to the action obtained by integrating the $B_{\mu\nu}$ field out, where the conformal Killing equation of c^μ is not necessary even in the later case.

Δ_4 is the conformally covariant 4th order operator [3],

$$\Delta_4 = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^\mu R)\nabla_\mu \ . \tag{18}$$

Why the number of the independent parameters is three is due to the fact that R^2 is not integrable w.r.t. conformal mode [3].

We consider the following invariant action including 4th order terms:

$$I(f,g) = I_4 + I_{LE} \ , \tag{19}$$

where

$$I_4 = \frac{d}{(4\pi)^2}\int d^4x\sqrt{g}R^2 \ , \tag{20}$$

$$I_{LE} = \frac{1}{(4\pi)^2}\int d^4x\sqrt{g}(-m^2R + \Lambda) + I_M(f,g) \ . \tag{21}$$

I_4 is the 4th order action with $d > 0$ which as well as the WZ action could be regarded as being a part of the measure. I_{LE} is the usual 2nd order action which describes low-energy physics. m^2 is the inverse of the gravitational constant and Λ is the cosmological constant. I_M is a matter action. Here, note that the presence of the $\phi\bar{F}$ term in the WZ action implies that the Weyl term, $\sqrt{g}F$ $(= \sqrt{\bar{g}}\bar{F})$, can be produced by expanding around a vacuum expectation value (VEV) of ϕ.

Let us define the perturbation around VEV of ϕ such that the Weyl term, $\frac{1}{t^2}\bar{F}$, is produced, where we introduce the dimensionless coupling constant, t, for the traceless mode and the $h^\mu{}_\nu$ field is replaced with $th^\mu{}_\nu$ in the combined action [2, 1]. Then, the integral region of ϕ is effectively restricted within the region $-\frac{1}{t} < \phi < \frac{1}{t}$. This perturbation theory seems to be well-defined; namely, it is expected that t is small enough [1] in comparison with recent numerical experiments [20].

Let us consider the regularized 1PI effective action, $\Gamma_{\rm eff}$. As discussed in [1], diffeomorphism invariance, namely, $\delta\Gamma_{\rm eff} = 0$ gives constraints on actions of gravitational fields as well as matter fields, which requires that 4D model must satisfy the following conditions:

- Matter fields must *conformally* couple to gravity.

- The coefficient, d, in 4th order action must be

$$d = \frac{1}{36}(2a + 2b + 3c) \ . \tag{22}$$

The second condition means that self-interactions of the conformal mode, namely the R^2 terms, cancel out in the combined action. Then, the three coefficients a, b and c, can be determined uniquely in the perturbation of the coupling, t, by requiring diffeomorphism invariance.[5]

These conditions are more precisely represented as that, in the regularized 1PI effective action, there is no non-local correction to the WZ action like $\phi \bar{\Box}^2 \log(-\bar{\Box})\phi$ and no non-local term, $\bar{R}\log(-\bar{\Box})\bar{R}$, which is associated to non-conformally invariant counterterm proportional to $\bar{R}^2$. This is the generalized form of the integrability condition discussed in [3]. Here, there are two remarks. The first is that the two types of non-local corrections considered here are related to each other by the background-metric independence for the conformal mode. The second is that vanishing of the $\bar{R}\log(-\bar{\Box})\bar{R}$ term does not always imply vanishing of the $\bar{R}^2$ divergences at higher loops.

The combined action, $\mathcal{I} = S + I$, including the gauge-fixing term and the FP ghost action, now becomes

$$\begin{aligned}\mathcal{I} = \frac{1}{(4\pi)^2}\int d^4x \Big[& \frac{1}{t^2}\bar{F} + a\bar{F}\phi + 2b\phi\bar{\Delta}_4\phi + b\Big(\bar{G} - \frac{2}{3}\bar{\Box}\bar{R}\Big)\phi \\ & + \frac{1}{36}(2a + 2b + 3c)\bar{R}^2 + \mathcal{L}_{GF+FP}\Big] + I_{LE}(f, g) \ . \end{aligned} \tag{23}$$

Here and below, we take the flat background $\hat{g}_{\mu\nu} = \delta_{\mu\nu}$. The gauge-fixing term is given by [21].

$$\mathcal{L}_{GF} = 2iB^\mu N_{\mu\nu}\chi^\nu - \zeta B^\mu N_{\mu\nu} B^\nu \ , \tag{24}$$

where $\chi^\nu = \partial^\lambda h^\nu{}_\lambda$ and $N_{\mu\nu}$ is a symmetric 2nd order operator. The corresponding ghost action becomes 4th order:

$$\mathcal{L}_{FP} = -2i\bar{c}^\mu N_{\mu\nu}\partial^\lambda \delta_{\mathbf{B}} h^\nu{}_\lambda \ , \tag{25}$$

[5] The coefficients depend on matter contents, but the sign of a (b) is negative-definite (positive-definite) at the zero-th order of t. c vanishes at this order. And also the sum of them, d, becomes positive.

where the BRST transformation of the traceless mode is given by

$$\delta_{\mathbf{B}} h^\mu{}_\nu = i\Big[\partial^\mu c_\nu + \partial_\nu c^\mu - \frac{1}{2}\delta^\mu{}_\nu \partial_\lambda c^\lambda + t c^\lambda \partial_\lambda h^\mu{}_\nu + \frac{t}{2} h^\mu{}_\lambda \Big(\partial_\nu c^\lambda - \partial^\lambda c_\nu\Big) + \frac{t}{2} h^\lambda{}_\nu \Big(\partial^\mu c_\lambda - \partial_\lambda c^\mu\Big) + \cdots\Big] . \qquad (26)$$

This is given by replacing ξ^μ/t in the transformation with the contravariant vector ghost field, ic^μ. The kinetic term of the ghost action then becomes t-independent. The BRST transformations for other fields are given by

$$\begin{aligned} &\delta_{\mathbf{B}}\phi = itc^\lambda\partial_\lambda\phi + i\frac{t}{4}\partial_\lambda c^\lambda , \\ &\delta_{\mathbf{B}}\bar{c}^\mu = B^\mu , \qquad \delta_{\mathbf{B}}B^\mu = 0 , \\ &\delta_{\mathbf{B}} c^\mu = itc^\lambda\partial_\lambda c^\mu . \end{aligned} \qquad (27)$$

The transformations, (26) and (27), are nilpotent. Using the BRST transformation, the gauge-fixing term and the FP ghost action can be written as $\mathcal{L}_{GF+FP} = 2i\delta_{\mathbf{B}}\{\bar{c}^\mu N_{\mu\nu}(\chi^\nu + \frac{i}{2}\zeta B^\nu)\}$ [17].

As in 2D quantum gravity, the BRST transformation of the "classical" action, $\mathcal{I}$, is not BRST-invariant:

$$\delta_B \mathcal{I} = \frac{it}{4(4\pi)^2}\int d^4x \partial_\lambda c^\lambda \Big[a\Big(\bar{F} + \frac{2}{3}\,\bar{\Box}\,\bar{R}\Big) + b\bar{G} + c\,\bar{\Box}\,\bar{R}\Big] . \qquad (28)$$

The BRST invariance is equivalent to diffeomorphism invariance, which is realized dynamically as mentioned before.

If the B^μ field is integrated out, this field is related to the energy-momentum tensor through the field equation for the traceless mode. As in 2D quantum gravity, it means that the nilpotence of the BRST transformation requires that the BRST transformation of the energy-momentum tensor vanishes at the quantum level.

Let us consider the long-distance physics of this model. The theory is asymptotically free, namely, $a < 0$ for the coupling of the traceless mode, t. Thus, one can drop the Weyl term at the long distance. On the other hand, we leave the other three kinetic terms of gravitational fields: $\phi\bar{\Delta}_4\phi$, $\bar{R}^2$ and $m^2 R$. Let us compute the degrees of freedom in this case. Since $\bar{R}^2 = -\chi^\mu\partial_\mu\partial_\nu\chi^\nu + o(h^3)$, the 2nd-order operator, $N_{\mu\nu}$, is proportional to

$-\partial_\mu\partial_\nu + m'^2\delta_{\mu\nu}$,[6] where m'^2 comes from the Einstein-Hilbert term, which is proportional to m^2. The ghost determinant is now given by $\det^{1/2}(-\partial_\mu\partial_\nu + m'^2\delta_{\mu\nu}) \det M^{GH}_{\mu\nu}$, where $M^{GH}_{\mu\nu}$ is the usual 2nd order ghost operator of diffeomorphism invariance given by applying the BRST transformation to χ^μ such that $\delta_B \chi_\mu|_{h=0} = M^{GH}_{\mu\lambda} c^\lambda$. Note that $\det(-\partial_\mu\partial_\nu + m'^2\delta_{\mu\nu}) = \det(-\Box + m'^2)|_{\text{a scalar}}$, so that the number of ghost degrees of freedom is $4+4+1=9$. Hence, the total degrees of freedom becomes $2\times 1 + 9 - 9 = 2$. Thus, the negative-metric state related to the conformal mode is removed by ghosts. Really, due to the form of the BRST transformation, the conformal mode can not be a BRST invariant asymptotic state [16].

This result is quite suggestive, because, if there is a mechanism to remove the Weyl term, it seems that the theory becomes unitary. Recall that an exact diffeomorphism invariance implies that the integral region of ϕ is unrestricted above and below, so that the Weyl term can be absorbed into the $\phi\bar{F}$ term in the WZ action, which is just the original theory (19) with (22) adding the WZ action. Thus, we expect that diffeomorphism invariance ensures the unitarity non-perturbatively.

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